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A METHODOLOGY FOR INVESTIGATING THE FACTORS AFFECTING THE OUTCOME OF A SUBMARINE VERSUS SUBMARINE ENCOUNTER

by

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AFFECTING THE OUTCOME OF A SUBMARINE

VERSUS SUBMARINE ENCOUNTER

by

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ABSTRACT

A quantitative analysis of encounters between two submarines was made in order to investigate the effects different factors have on the outcome of such encounters. A model was constructed with the use of a logic tree and a computer simulation of a submarine versus submarine encounter. The computer simulation was developed from the logic tree. The outcomes of the simulation were analysed using contingency table tests of independence, the theory of games of timing and a linear statistical model. The contingency table tests and the theory of games of timing demonstrated the relationship of range and elapsed time to the possible outcomes. The linear statistical model was used to obtain estimates of the effects that various own ship and target capabilities have on the outcome of the encounters.

TABLE OF CONTENTS

APPENDIX A. Logic Tree

APPENDIX B. Computer Simulation Flow Chart

APPENDIX C. Computer Program for the Simulation of a Submarine versus Submarine Encounter

APPENDIX D. Sample Output of the Simulation

APPENDIX E. Computer Program for the Linear Statistical Model

LIST OF TABLES

CHAPTER I

INTRODUCTION

Background

Improvement in combat force capabilities and effectiveness has been and continues to be one of the major objectives of the military planner. In fact, since the advent of sophisticated and costly weapons systems capable of practically unlimited destruction, this objective has assumed even more importance. Trial and error methods of determining needed improvements are costly and inefficient. Proposed changes could yield little improvement in capability at a high cost whereas, at the same or perhaps a lesser cost, much larger gains could be obtained in other areas. It has long been realized that if encounters between opposing forces could be formulated into a mathematical model then an analysis of the parameters constituting the model could point out the areas in which improvement is most needed and would be most meaningful.

Previously Developed Models

Several types of models have been developed and utilized in the past and are still evolving. Some of the more notable of these are the Lanchester's Equations, game theory (in particular, games of timing or stochastic duels), war gaming, Monte Carlo simulations and other manual or computerized simulations.

TRW Systems, Incorporated, of Redondo Beach, California, utilized a model describing an encounter between a submarine and a surface ship task force in order to investigate certain surface ship sonar systems and ASW weapon systems. The model proposed by this thesis has been developed along the same lines as the TRW simulation, but differs in several respects. The most obvious difference is that this model describes an encounter between two submarines. Whereas the real elapsed time of the encounter was important in the TRW investigation, it is not pertinent to this one. TRW was specifically interested in comparing various sonar and weapon systems while this paper has attempted to investigate other factors as well. Finally, while the TRW model utilized a manual evaluation of the logic tree, this thesis has adapted the logic tree evaluation to a computer simulation.

The Submarine Versus Submarine Problem

In order to construct a valid model, it is necessary to have an understanding of the different factors which compose or affect the progress of the encounter. The fire control problem for a submarine versus submarine encounter is very complex. Achieving an accurate solution is much more difficult than in a context where the antagonists can "see" each other either electronically or visually, thus making relatively accurate range information an integral part of the problem. As new weapons that are capable of extended ranges and new sonars having

long range detection and classification capabilities are developed, the doctrine and equipment utilized in obtaining a submarine fire control solution are continually being refined and modernized. The standard method of obtaining a solution is the bearings-only approach. In this type of an approach, bearing rate and estimated target speed are used to arrive at a best estimate of target course, speed and range. The bearings only approach has the characteristic that the accuracy of the solution is always in doubt although it tends to improve with an increase in the time that the contact is held.

Obviously, the elapsed time between initial detection of a target and launching a weapon will affect the outcome of the encounter. Current doctrine specifies that in order for a submarine to most effectively use its weapon and sonar capabilities and at the same time maximize its survivability, it should shoot at as long a range as possible. There are, of course, many factors other than just firing doctrine which contribute to the determination of the final results of an encounter. The sophistication and reliability of the weapon system, sonar system and fire control system are extremely important as well as the state of training and experience level of the crew in utilizing these systems. The design of the ship, its propulsion and control systems, will also have a marked impact on the results.

A snap shot, as the term is used in this paper, is basically a defensive tactic. It is employed when a ship has been attacked. In such a situation, a homing torpedo is launched in the direction from which the attack was launched or is expected.

Organization of the Remainder of the Thesis

The objective of this thesis is to propose a methodology for the investigation of the effects each of the various factors has on the outcome of a submarine versus submarine encounter. The fulfillment of this objective is accomplished by constructing a model which makes use of a logic tree and computer simulation. These are discussed in detail in Chapter II, along with all simplifying assumptions. The evaluation of the simulation's results are contained in Chapter III. Contingency tables, the theory of stochastic duels and a linear statistical model were used in that evaluation. The conclusions reached after the analysis, and recommendations for further study are contained in Chapter IV. The appendices contain the various diagrams, computer programs and outputs relating to this study.

CHAPTER II

APPROACH TO THE PROBLEM

The initial step in the accomplishment of the objective of this paper was to build a valid model of an encounter between two submarines. A logic tree was constructed to graphically represent the encounter. From that logic tree, a computer program was developed to simulate the encounter. The computer was utilized to provide a large sample number of encounters in order that a more meaningful analysis of the factors involved could be made.

The Logic Tree

The logic tree as illustrated in Appendix A is a connected directed graph composed of probabilistic nodes. The nodes represent the opportunity for the occurrence of the pertinent events constituting the encounter. Each event has a positive probability of occurring. Each node has two arcs incident from it. One arc represents the actual occurrence of the event while, if it does not take place, the other arc is followed. For this model, the occurrence of an event was assumed to be a Bernoulli trial. The path the encounter follows down the tree is determined by comparing the probability associated with the first node with a random number drawn from a uniform (0,1) distribution. If the random number is less than or equal to the nodal probability the event is said to have occurred and the appropriate arc is followed to the next

node. Likewise, if the random number is greater than the nodal probability the event did not occur and the opposite arc is followed. This procedure is repeated until the path reaches its final node at which time the encounter is terminated.

To aid in the analysis and evaluation of the outcomes, the tree was divided into two cases. Case one represents an encounter in which the target does not detect the presence of own ship until after own ship has launched a weapon. Case two is the converse of case one in that the target first detects own ship prior to own ship launching a weapon.

Definition of Events and Outcomes

PCDET - Probability target detects own ship prior to own ship launching a weapon.

PATT - Probability own ship makes a successful attack on the target.

PCA - Probability target will counterattack prior to own ship's attack.

PCAT - Probability the target's counterattack is successful.

PTAS - Probability target attempts a snap shot, given the target has been fired upon.

PTSS - Probability the target's snap shot is successful.

PSS - Probability that own ship makes a successful snap shot, given that own ship has been fired upon.

PRAT - Probability own ship makes a successful reattack on the target.

PEVAD1 - Probability target evades (breaks off the encounter) given he has been attacked.

PEVAD2 - Probability target evades given he detects own ship prior to being attacked.

DPAT - Probability of successful attack on the target given the target detects own ship prior to being attacked.

UPTS - Probability target makes a successful snapshot given the target detects own ship prior to being attacked.

Each path in the tree terminates with one of four possible outcomes of an encounter. These outcomes are:

IW - Target is sunk and own ship survives the encounter (hereafter referred to as a successful attack).

IL - Own ship is sunk and the target survives the encounter.

IDW - Both ships survive the encounter.

IDL - Both ships are sunk.

Computer Simulation

The flow chart from which the computer program was written is contained in Appendix B. It was written in FORTRAN IV and run on an IBM 360 series computer. The computer program comprises Appendix C. The total time for compilation and execution was about 2.7 minutes. During this time, a total of twenty-seven different combinations of own ship and target capabilities were involved in one thousand encounters for each of three regions, yielding a total of 81,000 encounters.

Parameters and Associated Capabilities

The values assigned to the nodal probabilities reflect the relative capabilities of the submarines in-

volved. By simply altering the parameter values, the model can be made to describe encounters between any two types of submarines. The values for PCDET are a measure of the target's sonar system capability while the values assigned to PEVAD1 and PEVAD2 are measures of the target's ship system capability. These parameters can also be thought of as containing some yardstick of own ship's capabilities. As an example, a large value for PCDET might indicate a noisy attacking ship instead of a particularly sophisticated target sonar installation. By the same token, large values for PEVAD1 and PEVAD2 could reflect an own ship sonar system that is not at peak performance or not being properly utilized.

Similarly, the other parameters can be considered to reflect the weapon and fire control systems capabilities of the two ships. Low probabilities of successful snap shots are caused by the fact that when a snap shot is fired, the fire control solution is usually incomplete if there is a solution at all. Also, the crew is under some stress due to the fact that they know a weapon is in the water heading toward them.

Assumptions

The encounter envisioned as being applicable to this model is that of a barrier submarine versus a transitor, although the simulation could be modified so that it could be applied to other types of encounters as well. Some events which might occur in such an encounter have

been omitted because they do not affect the outcome or because they are not pertinent to the objective of the simulation. Encounters where the target detects own ship and attacks prior to own ship detecting the target and situations where own ship has been trailing the target for an extended period of time have not been considered.

Each submarine was assumed to have a warload mix of weapons on board and it was assumed that the maximum number of weapons either ship could fire in any one encounter was two. This weapon mix encompasses all ranges applicable to the encounter. As with the other capabilities considered for the two ship's involved, the types of weapons assumed to be carried can be tailored to the type of encounter desired.

If neither ship is sunk, the encounter is assumed to be terminated. It should be realized that either one or both ships might continue to prosecute the encounter, however it is considered that the termination assumption is not detrimental to the objective of this study.

Range Considerations

As was stated earlier, the values assigned to the probabilities associated with each node reflect the relative capabilities of the two submarines involved. It is intuitively reasonable that these parameters are also functions of the range between the two ships and the time contact has been held. Accordingly, as the range decreases, the parameter values will change. This property

is compensated for in the model by dividing the encounter into three regions. The selection and definition of these regions was somewhat arbitrary. It would be possible to divide the encounter into several more regions if a more detailed analysis were required. However, the three chosen are representative of the profile of a typical encounter and therefore describe the encounter sufficiently for the purpose of this model. It should be emphasized that the choice of the regions is dependent on the encounter under consideration so that the physical areas contained in each particular region may vary between one type of encounter and another. In adapting this model to a specific study, selection and description of regions must be tailored to fit the objective of that study.

In this problem, region one describes that period of the encounter during which the transitor is first detected and classified. A fire control solution is obtained which would enable the launching of a weapon with a reasonable expectation of a successful kill. Refinement of the fire control solution coupled with decreasing range brings the encounter into region two. As more data is collected, the solution is continually refined and updated in this region. Region three starts when the fire control solution can no longer be significantly improved and it lasts down to the minimum weapon employment range.

There are both advantages and disadvantages in launching a weapon as early in the encounter as possible.

An unalerted target is more vulnerable than an alerted one. However, the later in the encounter an attack is launched, the more likely it is that the target counterdetects the attacking ship and presses an attack of its own. At the same time, however, a premature attack that is unsuccessful can alert a target which previously had no knowledge of the presence of another ship. These factors will be investigated by analysing the results of attacks delivered in each of the three regions detailed above.

Parameter Values

Throughout the study, own ship was assumed to detect the target first. The parameters which were varied in order to examine the effect of different capabilities are listed below.

PATT
1. Relatively constant over range
2. Decrease slightly with range
3. Decrease proportional to range

PCDET
1. Capability comparable to own ship
2. Capability slightly inferior to own ship
3. Completely inferior to own ship

PCAT
1. Well trained crew; Sophisticated equipment
2. Medium capability
3. Poor capability

The values of these parameters, as listed in Table I, are based on operational experience and are considered to be reasonable representations of the capabilities considered. The values for the parameters PSS, PRAT and DPAT reflect the same capabilities as those considered for PATT. The values for PCA, PTAS, PEVAD1 and PEVAD2 are derived from the capabilities listed for PCDET while PTSS and UPTS

TABLE I

VALUES OF NODAL PROBABILITIES

CAPABILITIES	REGION	PCDET	PATT	PCA	PCAT	PTAS	PTSS
1	1	.88	.71	.30	.50	.50	.18
1	2	.93	.73	.50	.60	.40	.23
1	3	.98	.75	.70	.70	.30	.28
2	1	.70	.55	.30	.30	.45	.13
2	2	.80	.65	.50	.40	.35	.18
2	3	.90	.75	.70	.50	.25	.23
3	1	.10	.25	.30	.10	.30	.08
3	2	.50	.50	.50	.20	.20	.13
3	3	.80	.75	.70	.30	.10	.18
		PSS	PRAT	PEVAD1	PEVAD2	DPAT	UPTS
1	1	.20	.81	.10	.15	.66	.20
1	2	.25	.83	.08	.25	.68	.25
1	3	.30	.85	.05	.35	.70	.30
2	1	.15	.65	.08	.10	.50	.15
2	2	.20	.75	.06	.20	.10	.20
2	3	.25	.85	.03	.30	.70	.25
3	1	.10	.35	.06	.05	.20	.10
3	2	.15	.60	.04	.15	.45	.15
3	3	.20	.85	.01	.25	.70	.20

reflect those of PCAT.

Measure of Effectiveness

There are several measures of effectiveness which could be considered in the evaluation of the simulation results. Exchange ratio between the two ships involved is the measure of effectiveness chosen. It is defined as the ratio of total target losses to total own ship losses. Other factors, including the four possible outcomes previously mentioned, have also been examined.

CHAPTER III

RESULTS OF THE SIMULATION

Qualitatively, the results of the computer simulation were reasonable. These results, in the form of exchange ratios and outcomes, are listed in Tables II through VI. In order to make meaningful use of these results, however, they must be capable of quantitative analysis. For the determination of the effect range and elapsed time have on the outcome, contingency table tests were made to determine the dependency characteristics between regions and outcomes. Then a brief comparison was made between the results of the simulation and results which might be obtained by the employment of the theory of games of timing. In the analysis of the effect of different own ship and target capabilities on the outcomes of encounters, a linear statistical model of full rank was used in an attempt to determine exactly what part is played by each capability.

Contingency Table Test

The tabulation of exchange ratios in Table II for given ship capabilities and regions leads to the immediate conclusion that as the elapsed time of the encounter increases, the exchange ratios become less favorable to own ship. It is intuitively reasonable to assume, from these results, that it would be advantageous to own ship to prosecute the attack in region one or as early in

TABLE II

TABULATION OF EXCHANGE RATIOS
vs.
REGIONS AND CAPABILITIES

PCDET	PCAT	PATT	REGION I	REGION II	REGION III
1	1	1	3.35	1.32	.80
1	1	2	3.30	1.55	.87
1	1	3	2.20	1.32	.74
1	2	1	4.32	2.25	1.18
1	2	2	4.94	2.46	1.05
1	2	3	3.62	1.93	1.05
1	3	1	7.74	3.54	1.97
1	3	2	9.42	3.52	1.77
1	3	3	7.29	4.18	1.73
2	1	1	4.05	1.94	1.02
2	1	2	4.08	2.29	.96
2	1	3	3.35	1.86	.92
2	2	1	5.06	2.65	1.29
2	2	2	5.99	2.88	1.45
2	2	3	6.05	2.66	1.28
2	3	1	7.87	4.64	2.10
2	3	2	10.75	4.38	2.09
2	3	3	8.93	4.71	2.51
3	1	1	9.24	3.45	1.19
3	1	2	10.42	3.39	1.22
3	1	3	9.76	2.98	1.25
3	2	1	9.15	4.53	1.45
3	2	2	11.71	4.65	1.54
3	2	3	11.46	5.60	1.59
3	3	1	8.45	6.09	2.46
3	3	2	12.54	7.20	2.72
3	3	3	16.64	7.51	2.77

TABLE III

TABULATION OF SUCCESSFUL ATTACKS

vs.

REGIONS AND CAPABILITIES

PCDET	PCAT	PATT	REGION I	REGION II	REGION III
1	1	1	569	361	247
1	1	2	540	401	278
1	1	3	345	356	267
1	2	1	565	419	284
1	2	2	535	434	274
1	2	3	334	384	278
1	3	1	577	451	356
1	3	2	566	417	325
1	3	3	343	407	316
2	1	1	611	460	296
2	1	2	587	497	306
2	1	3	381	419	309
2	2	1	627	470	335
2	2	2	600	486	353
2	2	3	375	435	349
2	3	1	653	483	362
2	3	2	612	510	369
2	3	3	390	464	393
3	1	1	816	617	360
3	1	2	761	588	379
3	1	3	483	529	408
3	2	1	818	641	357
3	2	2	745	624	390
3	2	3	454	570	396
3	3	1	808	643	427
3	3	2	767	604	442
3	3	3	455	588	436

TABLE IV

TABULATION OF BOTH SHIPS SUNK

vs.

REGIONS AND CAPABILITIES

PCDET	PCAT	PATT	REGION I	REGION II	REGION III
1	1	1	81	137	154
1	1	2	48	91	136
1	1	3	29	62	84
1	2	1	78	71	120
1	2	2	38	63	117
1	2	3	17	41	66
1	3	1	58	52	74
1	3	2	38	40	67
1	3	3	14	24	40
2	1	1	81	108	165
2	1	2	57	79	112
2	1	3	21	45	91
2	2	1	82	87	121
2	2	2	35	49	88
2	2	3	12	38	72
2	3	1	71	60	65
2	3	2	33	51	64
2	3	3	21	26	43
3	1	1	80	97	143
3	1	2	62	67	116
3	1	3	15	44	93
3	2	1	88	93	133
3	2	2	51	64	90
3	2	3	16	35	60
3	3	1	16	88	80
3	3	2	48	44	58
3	3	3	11	28	32

TABLE V

TABULATION OF BOTH SHIPS SURVIVE

vs.

REGIONS AND CAPABILITIES

PCDET	PCAT	PATT	REGION I	REGION II	REGION III
1	1	1	237	261	250
1	1	2	282	282	248
1	1	3	485	327	257
1	2	1	286	363	374
1	2	2	349	364	354
1	2	3	569	396	395
1	3	1	341	407	426
1	3	2	381	453	454
1	3	3	608	490	478
2	1	1	218	247	251
2	1	2	255	251	257
2	1	3	499	331	257
2	2	1	233	320	311
2	2	2	294	328	342
2	2	3	561	387	321
2	3	1	255	400	435
2	3	2	328	362	424
2	3	3	564	432	433
3	1	1	87	176	219
3	1	2	160	219	215
3	1	3	466	279	192
3	2	1	83	197	305
3	2	2	187	228	299
3	2	3	505	322	318
3	3	1	85	237	367
3	3	2	168	306	374
3	3	3	517	330	395

TABLE VI

TABULATION OF OWN SHIP SUNK, TARGET SURVIVES
vs.
REGIONS AND CAPABILITIES

PCDET	PCAT	PATT	REGION I	REGION II	REGION III
1	1	1	113	241	349
1	1	2	130	226	338
1	1	3	141	255	392
1	2	1	71	147	222
1	2	2	78	139	255
1	2	3	80	179	261
1	3	1	24	90	144
1	3	2	25	90	154
1	3	3	35	79	166
2	1	1	90	185	288
2	1	2	101	173	325
2	1	3	99	205	343
2	2	1	58	123	233
2	2	2	71	137	217
2	2	3	52	140	258
2	3	1	21	57	138
2	3	2	27	77	143
2	3	3	25	78	131
3	1	1	17	110	278
3	1	2	17	126	290
3	1	3	36	148	307
3	2	1	11	69	205
3	2	2	17	84	221
3	2	3	25	73	226
3	3	1	11	32	126
3	3	2	17	46	126
3	3	3	17	54	137

the encounter as possible. In order to support this conclusion, it would be desirable to show that the occurrence of the possible outcomes of the encounters are dependent on the region in which the attack is launched. Tests of independence in contingency tables as discussed in reference 4 provide a convenient means of examining these dependency properties. If an encounter was to be considered an experiment whose results could be classified by two attributes, namely region and outcome, then a contingency table would be an appropriate model. Such a model is illustrated below.

	SUCCESSFUL ATTACKS	BOTH SHIPS SUNK	BOTH SHIPS SURVIVE	OWN SHIP SUNK
REGION I	X_{11}	X_{12}	X_{14}	X_{14}
REGION II	X_{21}	X_{22}	X_{23}	X_{24}
REGION III	X_{31}	X_{32}	X_{33}	X_{34}

The X_{ij}'s represent the number of appropriate outcomes of the encounter while the random variable

$$\sum_{j=1}^{4} \sum_{i=1}^{3} \frac{\left[X_{ij} - n(X_{i}./n)(X._{j}/n)\right]^2}{n(x_{i}./n)(X._{j}/n)}$$

$X_{i}.$ = Sum of the elements of row i.

$X._{j}$ = Sum of the elements of column j.

n = Number of observations (encounters).

has an approximate Chi-square distribution with five degrees of freedom. If the value of the above expression is greater than the value in the Chi-square table for the desired significance level then the hypothesis that the out-

comes are independent of the regions is rejected at that significance level. Using the results of three thousand encounters for each of the twenty-seven different combinations of the capabilities, it was found that the hypothesis of independence was rejected at any level of significance tabulated. The conclusion is therefore drawn that the independence assumption should not be accepted and that the intuitive analysis of the table of exchange ratios is valid.

Games of Timing

The proposition that it is desirable to shoot as early in an encounter as possible is also borne out by the theory of games of timing, or duels, as discussed in reference 1. The type of encounter which is the subject of this study would be classified as a noisy duel with many bullets. In such a duel, the antagonists should shoot as soon as the following equation is satisfied:

$$P(t) = \frac{1}{m(t)+n(t)}$$

$P(t)$ is the accuracy function or probability of successful attack, while $m(t)$ and $n(t)$ are the number of weapons available on each ship for the particular encounter. Although this particular model assumes equal accuracies for both ships, it does provide a means for studying an encounter of this type. Assuming that each ship has at least two weapons, the value of $P(t)$ which satisfies the equation is

less than or equal to .25. This value occurs in region one for all of the encounters which were considered. An interesting study would be the application of the theory of stochastic duels, extended to include varying single shot kill probabilities as developed in reference 2, to this submarine versus submarine situation.

Linear Statistical Model

In order to study the effect that different own ship and target capabilities have on the outcome of an encounter, use was made of a linear statistical model of full rank. In this type of model, there is a known outcome assumed to be a random variable which is a function of p unknown parameters. If there are n outcomes then the known quantities form a matrix and the unknown parameters form a vector. Written in matrix notation, the linear model becomes:

$$Y = XB + e$$

Y is an nx1 vector of known outcomes.

X is an nxp matrix of known quantities. The rank X is p.

B is a px1 vector of unknown parameters.

ϵ is a px1 vector of error terms.

A least squares solution to this equation yields a best linear unbiased estimator of the vector B. Written in matrix form, this estimator, $\hat{B}$, is:

$$\hat{B} = (X'X)^{-1}X'Y$$

Returning to the objective of this study, it is obvious that if the outcomes of the encounters and the capabilities of the ships involved could be fitted into such a model, then the values for B should be good estimates of the relative effects which the various capabilities have on the outcomes.

Initial attempts at this method were unsuccessful. Five Y vectors, dimensioned 16x1, were examined using X matrices dimensioned 16x8 and 16x9. The five Y vectors corresponded to the possible outcomes of an encounter and the exchange ratios as detailed below:

Y_1 - Successful Attacks

Y_2 - Both Ships Sunk

Y_3 - Both Ships Survive

Y_4 - Own Ship Sunk, Target Survives

Y_5 - Exchange Ratios

The values of the components of the vectors Y_1 through Y_4 were divided by 1000 to give a relative frequency of occurrence for any given set of capabilities. Eighteen encounters were selected out of the eighty-one possible combinations of capabilities and regions. The encounters chosen included six different capability mixes with attacks occurring in each of the three regions. The dimension was reduced from eighteen to sixteen to make the X matrix nonsingular. The eight column vectors of the X matrix corresponded to the appropriate values for PCDET, PATT, PCAT, PSS, PEVAD1 and PEVAD2. The other nodal values

were not included because they formed linear combinations of the other columns and would have resulted in a singular matrix. In order to reduce the restrictions on the solutions a unit column vector was added to the X matrix bringing its total dimension to 16x9. The values for $\hat{B}$ arrived at by this computation were not satisfactory. They did not reflect the relationship which should exist between the parameters and the outcomes. This was borne out by the fact that when the matrix multiplication, $X\hat{B}$, was performed, it did not yield an estimate of the Y vector which could be correlated in any manner with the actual components of Y. The many unsuccessful attempts to fit the parameter and outcome values to the model indicated a conclusion that the factors affecting the outcome of an encounter do not have a linear relationship to that outcome.

However, there are three factors which were previously assumed to embody the capabilities reflected by the other parameter values. These three factors are PCDET, PATT and PCAT. Column vectors of these values plus a unit column vector were used to form a new X matrix. The element of B corresponding to the unit column vector could be considered to incorporate the terms deleted from the original X matrix so that the B_1 value reflects the cumulative effects of the deleted terms. Since the previous investigation into the effect that range has on the outcome of an encounter indicated that the optimum time to launch an attack was in region one, only that region was considered in

the new analysis. The linear model was then written:

$$Y = B_1\underline{1} + B_2X_2 + B_3X_3 + B_4X_4 + e$$

B_1, B_2, B_3, and B_4 are the elements of the B vector. The unit column vector is represented by $\underline{1}$, X_2 is the column vector of PCDET values, X_3 is the column vector of PATT values and X_4 is the column vector of PCAT values. The values for B resulting from these computations are shown in Table VII.

TABLE VII

VALUES OF B FOR THE DIFFERENT Y VECTORS

	Y_1	Y_2	Y_3	Y_4	Y_5
$\hat{B}_1$	.50281	.01544	.44448	.03727	8.36105
$\hat{B}_2$	-.30239	-.02100	.24811	.07527	-4.96775
$\hat{B}_3$	.47109	.10424	-.47537	-.09996	3.38144
$\hat{B}_4$	.00874	.00562	-.14913	.13477	.5.22542

The estimates of the Y vectors resulting from the multiplication $X\hat{B}$ coincided more closely with the actual values of Y than did the values obtained with the larger X matrix. In addition, nine sets of capabilities which were not used in the determination of $\hat{B}$ were substituted into the linear model. The estimates of Y coincided extremely well with the actual values of Y obtained from the simulation. The results of these computations are contained in Tables VIII and IX. This should indicate that a linear relationship does exist

TABLE VIII

RESULTS OF THE LINEAR STATISTICAL MODEL

Successful Attacks

Y'	= (.569	.540	.345	.611	.816	.565	.577	.627	.808)
$\hat{Y}'$	= (.576	.500	.359	.630	.811	.574	.572	.628	.808)
$\lvert Y'-\hat{Y}'\rvert$	= (.007	.040	.014	.019	.005	.009	.005	.001	.000)

Both Ships Sunk

Y'	= (.081	.048	.029	.081	.080	.078	.058	.082	.096)
$\hat{Y}'$	= (.074	.057	.026	.078	.090	.073	.072	.076	.088)
$\lvert Y'-\hat{Y}'\rvert$	= (.007	.009	.003	.003	.010	.005	.014	.006	.008)

Both Ships Survive

Y'	= (.237	.282	.485	.218	.087	.286	.341	.233	.085)
$\hat{Y}'$	= (.251	.327	.469	.206	.057	.281	.310	.236	.117)
$\lvert Y'-\hat{Y}'\rvert$	= (.014	.045	.016	.012	.030	.005	.031	.003	.032)

Own Ship Sunk, Target Survives

Y'	= (.113	.130	.141	.090	.107	.071	.024	.058	.011)
$\hat{Y}'$	= (.100	.116	.146	.086	.041	.073	.046	.059	.013)
$\lvert Y'-\hat{Y}'\rvert$	= (.013	.014	.005	.004	.024	.002	.022	.001	.024)

Exchange Ratios

Y'	= (3.35	3.30	2.20	4.05	9.24	4.32	7.74	5.06	8.45)
$\hat{Y}'$	= (3.78	3.24	2.22	4.67	7.65	4.82	5.87	5.72	9.74)
$\lvert Y'-\hat{Y}'\rvert$	= (.43	.06	.02	.62	1.59	.50	1.87	.66	1.29)

TABLE IX

COMPARISON OF OTHER VALUES OF Y

Successful Attacks

$Y' = (.535\ .334\ .566\ .587\ .381\ .600\ .375\ .653\ .818)$

$\hat{Y}' = (.498\ .357\ .497\ .555\ .413\ .553\ .412\ .626\ .810)$

$|Y'-\hat{Y}'| = (.037\ .023\ .069\ .032\ .032\ .047\ .037\ .027\ .008)$

Both Ships Sunk

$Y' = (.038\ .017\ .038\ .057\ .021\ .035\ .012\ .071\ .088)$

$\hat{Y}' = (.056\ .025\ .055\ .061\ .030\ .060\ .028\ .075\ .089)$

$|Y'-\hat{Y}'| = (.018\ .008\ .017\ .004\ .009\ .025\ .016\ .004\ .001)$

Both Ships Survive

$Y' = (.349\ .569\ .381\ .255\ .499\ .294\ .561\ .255\ .083)$

$\hat{Y}' = (.357\ .499\ .386\ .282\ .424\ .312\ .455\ .266\ .087)$

$|Y'-\hat{Y}'| = (.008\ .070\ .005\ .027\ .074\ .018\ .106\ .011\ .004)$

Own Ship Sunk, Target Survives

$Y' = (.078\ .080\ .025\ .101\ .099\ .071\ .052\ .021\ .011)$

$\hat{Y}' = (.089\ .119\ .062\ .102\ .132\ .075\ .105\ .032\ .014)$

$|Y'-\hat{Y}'| = (.011\ .039\ .037\ .001\ .033\ .004\ .053\ .011\ .003)$

Exchange Ratios

$Y' = (4.94\ 3.62\ 9.42\ 4.08\ 3.35\ 5.99\ 6.05\ 7.87\ 9.15)$

$\hat{Y}' = (4.28\ 3.27\ 5.33\ 4.13\ 3.12\ 5.18\ 4.16\ 6.76\ 8.70)$

$|Y'-\hat{Y}'| = (\ .66\ \ .35\ 4.09\ \ .05\ \ .23\ \ .81\ 1.89\ 1.10\ \ .45)$

between outcomes and the factors affecting them, and, if that relationship could be found, a meaningful analysis of the factors involved could be made.

The calculations associated with the linear statistical model were made on an IBM 360 Series computer. The program for those calculations is contained in Appendix G.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This study has attempted to model an encounter between two submarines and to quantitatively analyse the factors involved. The logic tree and the computer simulation together form a representation of encounters which can be made applicable to a variety of own ship and target capabilities. A large sample of encounters can thus be analysed.

The results of the model indicate the desirability of prosecuting an attack in region one instead of regions two or three. The demonstration of the dependency of the outcomes on the region in which the attack is launched lend added significance to this conclusion, thus supporting present tactical doctrine. This support also tends to add credence to the validity of the model.

The use of the linear statistical model and the results obtained from it were disappointing at first. However, a linear unbiased estimator which could accurately estimate the original Y vector was obtained using a reduced size X matrix. This final estimator did reflect reasonable representations of the effects the different factors should have on the outcomes of the encounters. Further refinement of this technique should yield a method for obtaining a meaningful analysis of the effect proposed systems would have on ship capabilities.

Recommendations

The model proposed in this thesis is not intended to replace other types of models now in use, nor is it designed to solve all problems. However, for problems involving two opposing forces and whose characteristics can be applicably quantified, this basic model presents a methodology which could be an effective tool in the hands of an analyst.

Application of this model to an actual situation immediately poses the question as to where values for the nodal probabilities can be obtained or in what manner can they be determined? The simulation and the analysis of its results become an academic exercise if they cannot be compared with actual results and applied to actual problems. Fortunately, there is a wealth of information contained in the reports of exercises conducted by various submarine commands and by SUBMARINE DEVELOPMENT GROUP TWO. Since the subject of this thesis was to propose and develop a model, actual exercise data was not used, although the specific values desired could be extracted from such exercise data and therby apply the model to a "real world" situation.

This study has pointed to several areas where further development on the thesis level should prove to be fruitful. These areas are detailed below:

1. This model should be applicable to a variety of types of encounters. One that immediately comes

to mind is an aircraft versus aircraft encounter. A further extension would be an attempt to apply it to encounters between opposing forces where each force is composed of multiple units.

2. Further development of the linear statistical model is needed to obtain a best linear unbiased estimator which more closely reflects the effects all of the factors have in determining the outcome of an encounter.

3. According to the Central Limit Theorem, the large number of observations enable the assumption to be made that the e_{ij}'s of the linear statistical model are normally distributed random variables. Under this assumption, hypothesis tests on and confidence intervals for the values of $\hat{B}$ can be obtained. Such statistical tests should be made in order to gain more insight into the problem.

4. An attempt to apply the theory of stochastic duels with varying single shot kill probability, as detailed in reference 2, to the submarine versus submarine encounter should also prove to be profitable.

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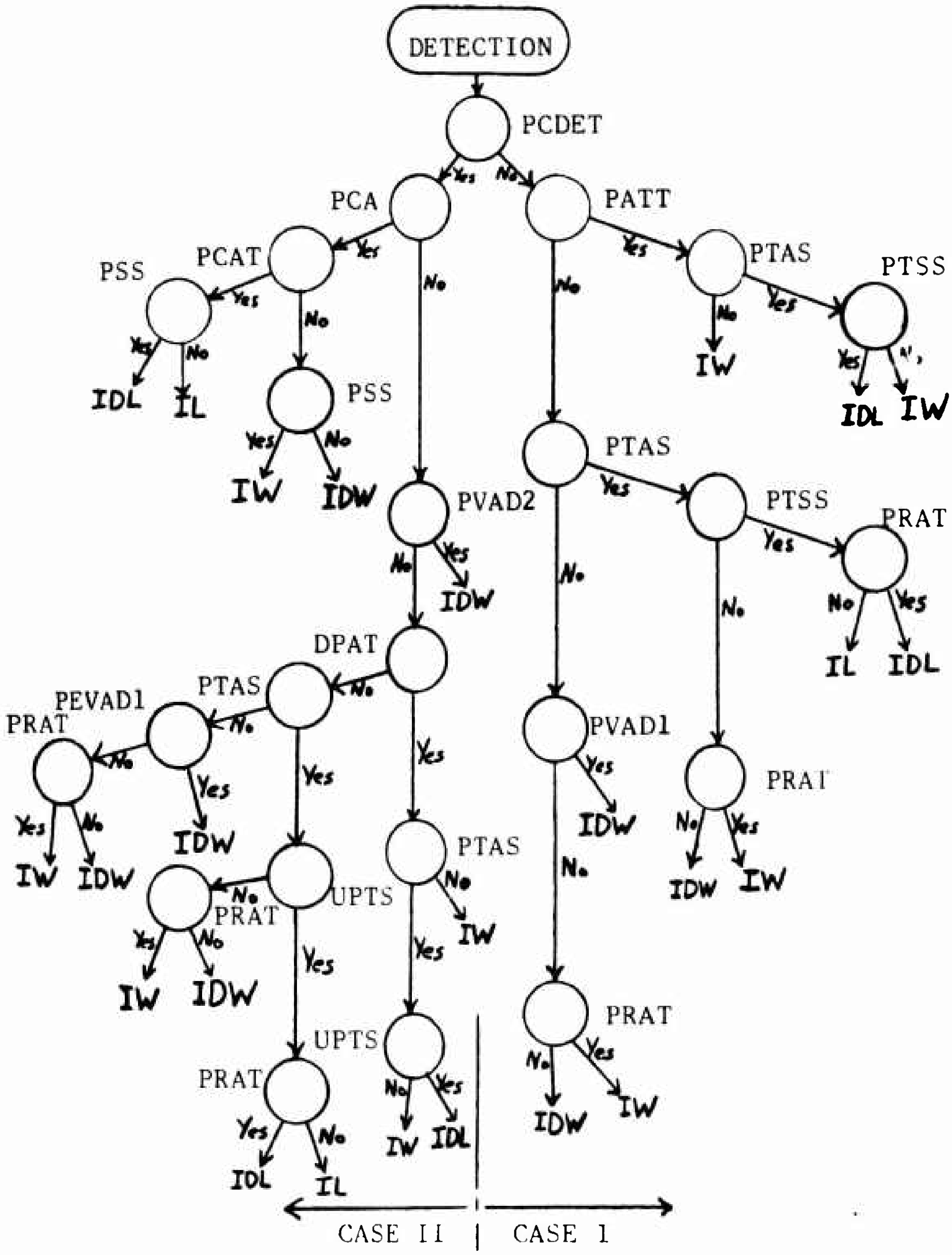
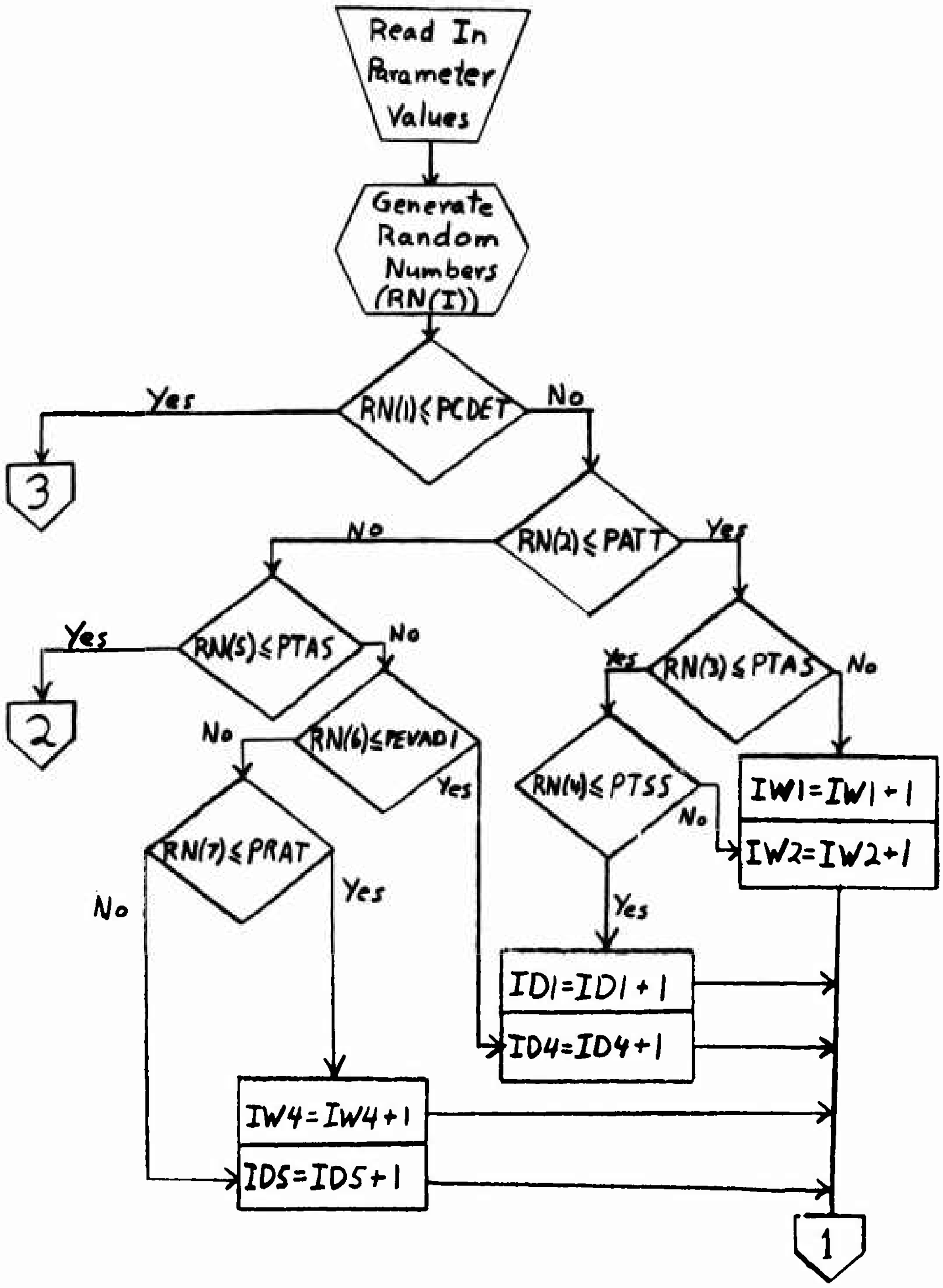
APPENDIX A

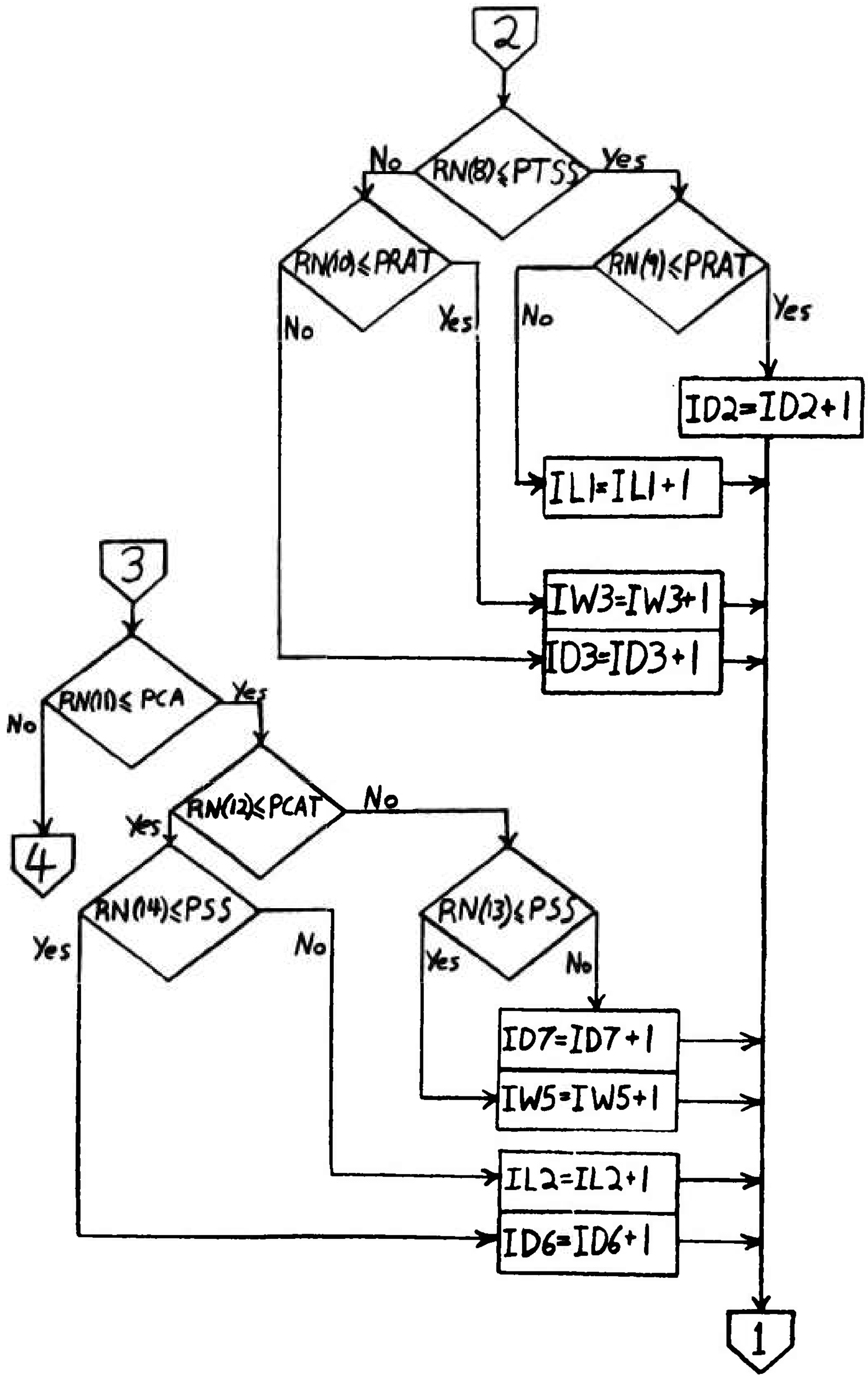
LOGIC TREE

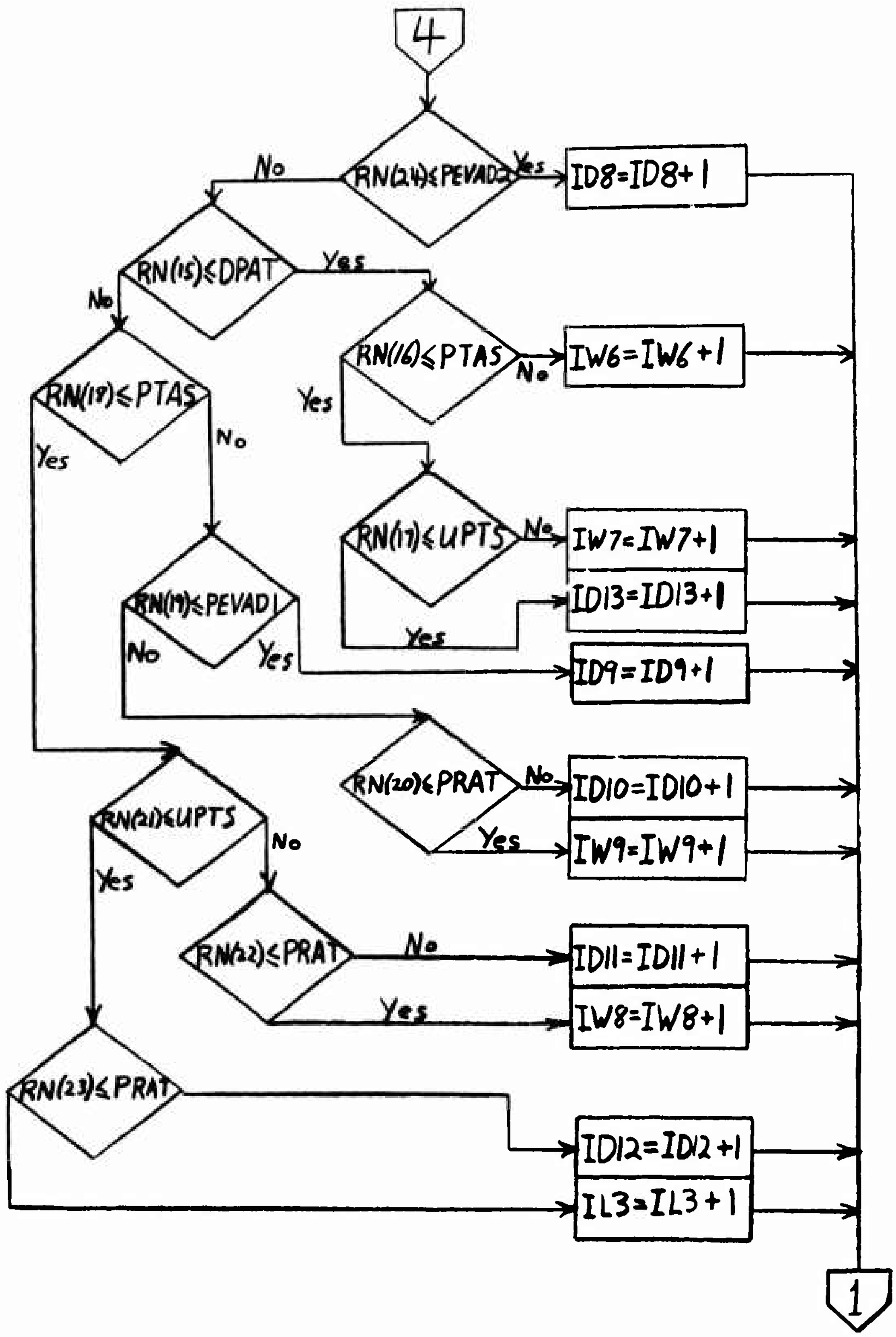


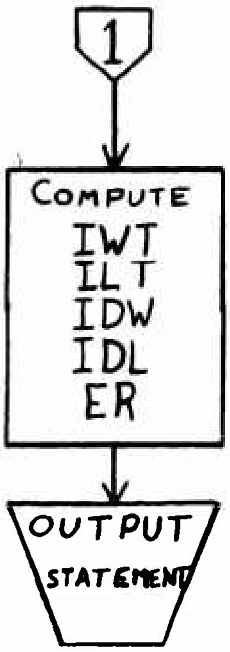
APPENDIX B

COMPUTER SIMULATION FLOW CHART




2
RN(8)≤PTSS
No
Yes
RN(10)≤PRAT
RN(9)≤PRAT
No
Yes
No
Yes
ID2=ID2+1
IL1=IL1+1
IW3=IW3+1
ID3=ID3+1
3
RN(11)≤PCA
No
Yes
4
RN(12)≤PCAT
Yes
No
RN(14)≤PSS
RN(13)≤PSS
Yes
No
Yes
No
ID7=ID7+1
IW5=IW5+1
IL2=IL2+1
ID6=ID6+1
1


4
RN(24)≤PEVAD2
No
Yes
ID8=ID8+1
RN(15)≤DPAT
Yes
No
RN(16)≤PTAS
No
IW6=IW6+1
Yes
RN(18)≤PTAS
Yes
No
RN(17)≤UPTS
No
IW7=IW7+1
ID13=ID13+1
Yes
RN(19)≤PEVAD1
No
Yes
ID9=ID9+1
RN(20)≤PRAT
No
ID10=ID10+1
Yes
IW9=IW9+1
RN(21)≤UPTS
No
Yes
RN(22)≤PRAT
No
ID11=ID11+1
Yes
IW8=IW8+1
RN(23)≤PRAT
ID12=ID12+1
IL3=IL3+1
1


1
COMPUTE
IWT
ILT
IDW
IDL
ER
OUTPUT
STATEMENT

APPENDIX C

COMPUTER PROGRAM FOR THE SIMULATION OF A SUBMARINE VERSUS SUBMARINE ENCOUNTER

```
      DIMENSION PCDET(3,3),PATT(3,3),PCA(3,3),PCAT(3,3),
     1PTAS(3,3),PTSS(3,3),PSS(3,3),PRAT(3,3),PEVAD1(3,3),
     1PEVAD2(3,3),DPAT(3,3),UPTS(3,3),ID(15),IW(15),
     1IL(15),RN(25),ER(9,3,3)
      DUM=URN(0)
      IK=0
      JK=0
      READ(5,1 )((PCDET(I,J),J=1,3),I=1,3)
      READ(5,2 )((PATT  (I,J),J=1,3),I=1,3)
      READ(5,3 )((PCA   (I,J),J=1,3),I=1,3)
      READ(5,4 )((PCAT  (I,J),J=1,3),I=1,3)
      READ(5,5 )((PTAS  (I,J),J=1,3),I=1,3)
      READ(5,6 )((PTSS  (I,J),J=1,3),I=1,3)
      READ(5,7 )((PSS   (I,J),J=1,3),I=1,3)
      READ(5,8 )((PRAT  (I,J),J=1,3),I=1,3)
      READ(5,9 )((PEVAD1(I,J),J=1,3),I=1,3)
      READ(5,10)((PEVAD2(I,J),J=1,3),I=1,3)
      READ(5,11)((DPAT   (I,J),J=1,3),I=1,3)
      READ(5,12)((UPTS   (I,J),J=1,3),I=1,3)
    1 FORMAT(9F3.2)
    2 FORMAT(9F3.2)
    3 FORMAT(9F3.2)
    4 FORMAT(9F3.2)
    5 FORMAT(9F3.2)
    6 FORMAT(9F3.2)
    7 FORMAT(9F3.2)
    8 FORMAT(9F3.2)
    9 FORMAT(9F3.2)
   10 FORMAT(9F3.2)
   11 FORMAT(9F3.2)
   12 FORMAT(9F3.2)
      WRITE(6,13)
   13 FORMAT(1H1)
C     L IS THE INDEX FOR VARIATION OF PCDET CAPABILITY.
C     K IS THE INDEX FOR VARIATION OF PCAT CAPABILITY.
C     I IS THE INDEX FOR VARIATION OF PATT CAPABILITY.
C     J IS THE INDEX FOR VARIATION OF REGION IN WHICH
C     ATTACK IS LAUNCHED.
      DO  5000 L=1,3
      DO 4000 K=1,3
      JK=JK+1
      DO 3000 I=1,3
      DO 2000 J=1,3
      DO 20 I1=1,15
      ID(I1)=0
      IL(I1)=0
   20 IW(I1)=0
      IDT=0
      ILT=0
      IWT=0
      IDT1=0
      IDT2=0
      ILT1=0
      ILT2=0
      IWT1=0
      IWT2=0
      ICD=0
      DO 90 J1=1,1000
C     GENERATION OF 25 RANDOM NUMBERS FOR EACH ENCOUNTER
      DO 21 I1=1,25
      RN(I1)=URN(1)
   21 CONTINUE
C     ENCOUNTER STARTS
      IF(RN(1).LE.PCDET(L,J))GO TO 50
C     CASE I
```

```
      IF(RN(2).LE.PATT(I,J))GO TO 22
      IF(RN(5).LE.PTAS(I,J))GO TO 24
      IF(RN(6).LE.PEVAD1(I,J))GO TO 27
      IF(RN(7).LE.PRAT(I,J))GO TO 28
      ID(5)=ID(5)+1
      GO TO 90
   22 IF(RN(3).LE.PTAS(I,J))GO TO 23
      IW(1)=IW(1)+1
      GO TO 90
   23 IF(RN(4).LE.PTSS(I,J))GO TO 31
      IW(2)=IW(2)+1
      GO TO 90
   24 IF(RN(8).LE.PTSS(I,J))GO TO 25
      IF(RN(10).LE.PRAT(I,J))GO TO 26
      ID(3)=ID(3)+1
      GO TO 90
   25 IF(RN(9).LE.PRAT(I,J))GO TO 32
      IL(1)=IL(1)+1
      GO TO 90
   26 IW(3)=IW(3)+1
      GO TO 90
   27 ID(4)=ID(4)+1
      GO TO 90
   28 IW(4)=IW(4)+1
      GO TO 90
   31 ID(1)=ID(1)+1
      GO TO 90
   32 ID(2)=ID(2)+1
      GO TO 90
C     CASE II
   50 ICD=ICD+1
      IF(RN(11).LE.PCA(I,J))GO TO 70
      IF(RN(24).LE.PEVAD2(I,J))GO TO 62
      IF(RN(15).LE.DPAT(I,J))GO TO 58
      IF(RN(18).LE.PTAS(I,J))GO TO 54
      IF(RN(19).LE.PEVAD1(I,J))GO TO 52
      IF(RN(20).LE.PRAT(I,J))GO TO 53
      ID(10)=ID(10)+1
      GO TO 90
   52 ID(9)=ID(9)+1
      GO TO 90
   53 IW(9)=IW(9)+1
      GO TO 90
   54 IF(RN(21).LE.UPTS(I,J))GO TO 55
      IF(RN(22).LE.PRAT(I,J))GO TO 57
      ID(11)=ID(11)+1
      GO TO 90
   55 IF(RN(23).LE.PRAT(I,J))GO TO 56
      IL(3)=IL(3)+1
      GO TO 90
   56 ID(12)=ID(12)+1
      GO TO 90
   57 IW(8)=IW(8)+1
      GO TO 90
   58 IF(RN(16).LE.PTAS(I,J))GO TO 59
      IW(6)=IW(6)+1
      GO TO 90
   59 IF(RN(17).LE.UPTS(I,J))GO TO 60
      IW(7)=IW(7)+1
      GO TO 90
   60 ID(13)=ID(13)+1
      GO TO 90
   62 ID(8)=ID(8)+1
      GO TO 90
   70 IF(RN(12).LE.PCAT(K,J))GO TO 71
```

```
      IF(RN(13).LE.PSS(I,J))GO TO 73
      ID(7)=ID(7)+1
      GO TO 90
   71 IF(RN(14).LE.PSS(I,J))GO TO 72
      IL(2)=IL(2)+1
      GO TO 90
   72 ID(6)=ID(6)+1
      GO TO 90
   73 IW(5)=IW(5)+1
   90 CONTINUE
      DO 91 K1=1,5
   91 IDT1=IDT1+ID(K1)
      DO 92 K1=6,13
   92 IDT2=IDT2+ID(K1)
      DO 93 K1=1,4
   93 IWT1=IWT1+IW(K1)
      DO 94 K1=5,9
   94 IWT2=IWT2+IW(K1)
      ILT1=IL(1)
      ILT2=IL(2)+IL(3)
      IDL1=ID(1)+ID(2)
      IDL2=ID(6)+ID(12)+ID(13)
      IDW2=ID(7)+ID(8)+ID(9)+ID(10)+ID(11)
      IDW1=ID(3)+ID(4)+ID(5)
      IDT=IDT1+IDT2
      ILT=ILT1+ILT2
      IWT=IWT1+IWT2
      ISUM=ID(1)+ID(2)+ID(6)+ID(12)+ID(13)
      SUM1=IWT+ISUM
      SUM2=ILT+ISUM
      EXR=(SUM1)/(SUM2)
      IDL=ISUM
      IDW=IDT-ISUM
      ER(JK,I,J)=EXR
      WRITE(6,200)J,I,K,L
      WRITE(6,201)
      WRITE(6,202)IW(1)
      WRITE(6,203)IW(2)
      WRITE(6,204)IW(3)
      WRITE(6,205)IW(4)
      WRITE(6,206)ID(1)
      WRITE(6,207)ID(2)
      WRITE(6,208)ID(3)
      WRITE(6,209)ID(4)
      WRITE(6,210)ID(5)
      WRITE(6,211)IL(1)
      WRITE(6,212)
      WRITE(6,213)IW(5)
      WRITE(6,214)IW(6)
      WRITE(6,215)IW(7)
      WRITE(6,216)IW(8)
      WRITE(6,217)IW(9)
      WRITE(6,218)ID(6)
      WRITE(6,219)ID(7)
      WRITE(6,220)ID(8)
      WRITE(6,221)ID(9)
      WRITE(6,222)ID(10)
      WRITE(6,223)ID(11)
      WRITE(6,224)ID(12)
      WRITE(6,225)ID(13)
      WRITE(6,226)IL(2)
      WRITE(6,227)IL(3)
      WRITE(6,234)IWT
      WRITE(6,235)IDL
      WRITE(6,2351)IDW
```

```
      WRITE(6,236)ILT
      WRITE(6,237)ICD
      WRITE(6,238) EXR
      WRITE(6,239)
  200 FORMAT(15X,6HREGION,I3,5X,4HPATT,I3,5X,4HPCAT,I3,5X,
     15HPCDET,I3/)
  201 FORMAT(15X,6HCASE I,/)
  202 FORMAT(15X,30HSUCCESSFUL ATTACKS (1)          =,I4/)
  203 FORMAT(15X,30HSUCCESSFUL ATTACKS (2)          =,I4/)
  204 FORMAT(15X,30HSUCCESSFUL ATTACKS (3)          =,I4/)
  205 FORMAT(15X,30HSUCCESSFUL ATTACKS (4)          =,I4/)
  206 FORMAT(15X,30HBOTH SHIPS SUNK (1)             =,I4/)
  207 FORMAT(15X,30HBOTH SHIPS SUNK (2)             =,I4/)
  208 FORMAT(15X,30HBOTH SHIPS SURVIVE (1)          =,I4/)
  209 FORMAT(15X,30HBOTH SHIPS SURVIVE (4)          =,I4/)
  210 FORMAT(15X,30HBOTH SHIPS SURVIVE (5)          =,I4/)
  211 FORMAT(15X,30HOWN SHIP SUNK (1)               =,I4/)
  212 FORMAT(15X,7HCASE II,/)
  213 FORMAT(15X,30HSUCCESSFUL ATTACKS (5)          =,I4/)
  214 FORMAT(15X,30HSUCCESSFUL ATTACKS (6)          =,I4/)
  215 FORMAT(15X,30HSUCCESSFUL ATTACKS (7)          =,I4/)
  216 FORMAT(15X,30HSUCCESSFUL ATTACKS (8)          =,I4/)
  217 FORMAT(15X,30HSUCCESSFUL ATTACKS (9)          =,I4/)
  218 FORMAT(15X,30HBOTH SHIPS SUNK (6)             =,I4/)
  219 FORMAT(15X,30HBOTH SHIPS SURVIVE (7)          =,I4/)
  220 FORMAT(15X,30HBOTH SHIPS SURVIVE (8)          =,I4/)
  221 FORMAT(15X,30HBOTH SHIPS SURVIVE (9)          =,I4/)
  222 FORMAT(15X,30HBOTH SHIPS SURVIVE (10)         =,I4/)
  223 FORMAT(15X,30HBOTH SHIPS SURVIVE (11)         =,I4/)
  224 FORMAT(15X,30HBOTH SHIPS SUNK (12)            =,I4/)
  225 FORMAT(15X,30HBOTH SHIPS SUNK (13)            =,I4/)
  226 FORMAT(15X,30HOWN SHIP SUNK (2)               =,I4/)
  227 FORMAT(15X,30HOWN SHIP SUNK (3)               =,I4/)
  234 FORMAT(15X,40HTOTAL SUCCESSFUL ATTACKS
     1                 =,I4/)
  235 FORMAT(15X,40HTOTAL BOTH SHIPS SUNK
     1                   =,I4/)
 2351 FORMAT(15X,40HTOTAL BOTH SHIPS SURVIVE
     1                =,I4/)
  236 FORMAT(15X,40HTOTAL OWN SHIP LOSS,TARGET SURVIVES
     1      =,I4/)
  237 FORMAT(15X,40HTOTAL OWN SHIP DETECTED BEFORE ATTACK
     1    =,I4/)
  238 FORMAT(15X,17HEXCHANGE RATIO = ,F5.2,2X,4HTO 1)
  239 FORMAT(1H1)
 2000 CONTINUE
 3000 CONTINUE
 4000 CONTINUE
 5000 CONTINUE
      WRITE(6,6001)
 6001 FORMAT(1H1,43X,15HEXCHANGE RATIOS//)
      DO 6008 L=1,3
      DO 6006 K=1,3
      IK=IK+1
      WRITE(6,6003)L,K
      WRITE(6,6004)
      DO 6002 I=1,3
      WRITE(6,6005)I,(ER(IK,I,J),J=1,3)
 6002 CONTINUE
 6003 FORMAT(40X,5HPCDET,I3,10X,4HPCAT,I3/)
 6004 FORMAT(15X,6HREGION,13X,1H1,13X,1H2,13X,1H3,/)
 6005 FORMAT(15X,4HPATT,I3,9X,F5.2,9X,F5.2,9X,F5.2,10X,
     14HTO 1/)
 6006 CONTINUE
 6008 CONTINUE
```

APPENDIX D

SAMPLE OUTPUT OF SIMULATION

REGION 1	PATT 1	PCAT 1	PCDET 1

CASE I

SUCCESSFUL ATTACKS (1)	= 42
SUCCESSFUL ATTACKS (2)	= 36
SUCCESSFUL ATTACKS (3)	= 11
SUCCESSFUL ATTACKS (4)	= 14
BOTH SHIPS SUNK (1)	= 7
BOTH SHIPS SUNK (2)	= 4
BOTH SHIPS SURVIVE (3)	= 0
BOTH SHIPS SURVIVE (4)	= 0
BOTH SHIPS SURVIVE (5)	= 2
OWN SHIP SUNK (1)	= 0

CASE II

SUCCESSFUL ATTACKS (5)	= 33
SUCCESSFUL ATTACKS (6)	=180
SUCCESSFUL ATTACKS (7)	=149
SUCCESSFUL ATTACKS (8)	= 47
SUCCESSFUL ATTACKS (9)	= 57
BOTH SHIPS SUNK (6)	= 24
BOTH SHIPS SURVIVE (7)	=107
BOTH SHIPS SURVIVE (8)	= 91
BOTH SHIPS SURVIVE (9)	= 10
BOTH SHIPS SURVIVE (10)	= 16

BOTH SHIPS SURVIVE (11) = 11

BOTH SHIPS SUNK (12) = 15

BOTH SHIPS SUNK (13) = 31

OWN SHIP SUNK (2) = 109

OWN SHIP SUNK (3) = 4

TOTAL SUCCESSFUL ATTACKS = 569

TOTAL BOTH SHIPS SUNK = 81

TOTAL BOTH SHIPS SURVIVE = 237

TOTAL OWN SHIP LOSS, TARGET SURVIVES = 113

TOTAL OWN SHIP DETECTED BEFORE ATTACK = 884

EXCHANGE RATIO = 3.35 TO 1

APPENDIX E

COMPUTER PROGRAM FOR THE LINEAR STATISTICAL MODEL

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y(16),X(16,9),XT(9,16),XTX(9,9),STO(16,16),
     1XTXI(9,9),XS(16),SIG(9),YHAT(16),YDIF(16),BH(9)
      WRITE (6,40)
   40 FORMAT (1H1)
      DO 100 KK=1,5
      N=4
      N1=4
      N2=9
      SUMX=0
      DO 10 I=1,N2
      READ(5,11) Y(I),(X(I,J),J=1,N1)
   11 FORMAT(10F4.2)
   10 CONTINUE
      DO 20 I=1,N1
      DO 21 J=1,N2
      XT(I,J)=X(J,I)
   21 CONTINUE
   20 CONTINUE
      DO 1 I=1,N1
      DO 3 I1=1,N1
      DO 2 J=1,N2
      SUMX=SUMX+(XT(I,J)*X(J,I1))
    2 CONTINUE
      XTX(I,I1)=SUMX
      SUMX=0
    3 CONTINUE
    1 CONTINUE
      CALL GAUSS3(N,EPS,XTX,XTXI,KER,9)
      IF(KER.NE.2)GO TO 61
      WRITE(6,62)
   62 FORMAT(15X,8HSINGULAR)
      STOP
   61 SUMX=0
      DO 50 J=1,N2
      WRITE(6,70) (XT(I,J),I=1,N1)
   70 FORMAT(5X,4F8.4//)
   50 CONTINUE
      DO 51 I=1,N1
      WRITE(6,71) (XTX(I,J),J=1,N1)
   71 FORMAT(5X,4F13.3//)
51    CONTINUF
      DO 52 I=1,N1
      WRITE(6,72) (XTXI(I,J),J=1,N1)
   72 FORMAT(20X,4F20.10//)
   52 CONTINUE
      DO 4 I2=1,N1
      DO 5 I3=1,N2
      DO 6 I4=1,N1
      SUMX=SUMX+(XTXI(I2,I4)*XT(I4,I3))
    6 CONTINUE
      STO(I2,I3)=SUMX
      SUMX=0
    5 CONTINUE
    4 CONTINUE
      SUNX=0
      DO 7 I5=1,N1
      DO 8 I6=1,N2
      SUNX=SUNX+(STO(I5,I6)*Y(I6))
    8 CONTINUE
      BH(I5)=SUNX
      SUNX=0
    7 CONTINUE
      YTY=0
      BXY=0
```

```
      DO 30 I=1,N2
      YTY=YTY+(Y(I)*Y(I))
   30 CONTINUE
      SUMX=0
      DO 31 J=1,N2
      DO 32 I=1,N1
      SUMX=SUMX+(BH(I)*XT(I,J))
   32 CONTINUE
      XS(J)=SUMX
      SUMX=0
   31 CONTINUE
      DO 33 I=1,N2
      BXY=BXY+(XS(I)*Y(I))
   33 CONTINUE
      SUMX=YTY-BXY
      D=N2-N1
      SH=(SUMX)/(D)
      DO 34 I=1,N1
      SIG(I)=(XTXI(I,I))*(SH)
   34 CONTINUE
      WRITE(6,12)
   12 FORMAT(25X,8HBETA HAT,25X,9HSIGMA HAT//)
      DO 25 J=1,N1
      WRITE(6,26)BH(J),SIG(J)
   26 FORMAT(15X,F30.15,5X,F30.15//)
   25 CONTINUE
      DO 42 I=1,N2
      SUMX=0
      DO 41 J=1,N1
      SUMX=SUMX+(BH(J)*X(I,J))
   41 CONTINUE
      YHAT(I)=SUMX
      YDIF(I)=Y(I)-YHAT(I)
   42 CONTINUE
      WRITE(6,43)
   43 FORMAT(15X,4HY(I),8X,7HYHAT(I),5X,7HYDIF(I))
      DO 45 I=1,N2
      WRITE(6,44) Y(I),YHAT(I),YDIF(I)
   44 FORMAT(13X,F12.10,2X,F12.10,2X,F12.10//)
   45 CONTINUE
      DO 80 I=1,N2
      READ(5,81)Y(I),(X(I,J),J=1,N1)
   81 FORMAT(5F4.2)
   80 CONTINUE
      DO 83 I=1,N2
      SUMX=0
      DO 84 J=1,N1
      SUMX=SUMX+(BH(J)*X(I,J))
   84 CONTINUE
      YHAT(I)=SUMX
      YDIF(I)=Y(I)-YHAT(I)
   83 CONTINUE
      WRITE(6,85)
   85 FORMAT(15X,4HY(I),8X,7HYHAT(I),5X,7HYDIF(I))
      DO 86 I=1,N2
      WRITE(6,87) Y(I),YHAT(I),YDIF(I)
   87 FORMAT(13X,F12.10,2X,F12.10,2X,F12.10//)
   86 CONTINUE
      WRITE(6,27)
   27 FORMAT (1H1)
  100 CONTINUE
      RETURN
      END
```

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13. ABSTRACT

A quantitative analysis of encounters between two submarines was made in order to investigate the effects different factors have on the outcome of such encounters. A model was constructed with the use of a logic tree and a computer simulation of a submarine versus submarine encounter. The computer simulation was developed from the logic tree. The outcomes of the simulation were analysed using contingency table tests of independence, the theory of games of timing and a linear statistical model. The contingency table tests and the theory of games of timing demonstrated the relationship of range and elapsed time to the possible outcomes. The linear statistical model was used to obtain estimates of the effects that various own ship and target capabilities have on the outcome of the encounters.

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